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The Vertice-Centered Metric Topologies Generated From the Connected Undirected Graphs

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Abstract

Graph theory is used as a way of specifying relationships among a collection of items. As a result of this, the theory is important for many fields from science to liberal arts. Recently, topological structure of the graphs is an interesting research topic. In this paper, we study topological structures of the connected undirected graphs. We firstly define a metric on a connected undirected graph using the distance between vertices of the graph. We generate a vertice-centered metric topology on vertices set of a connected undirected graph using this metric. Moreover, we study some properties of this topology. Finally, the vertice-centered metric topologies on vertices set of certain graphs are studied.

Keywords: Graph theory; metric topology of a graph; metric on a graph.

1 Introduction

Graph theory was presented by Leonhard Euler to solution of Königsberg Bridge Problem in 1736. Graphs are used to determine relationship among a collection of objects. As a result of this, the theory is used effectively in many fields from science to art sciences. Because rough sets and graphs are also based relational combinations, the topological structures of rough sets and relation between rough sets and graphs are studied by some researchers [5, 9]. Recently, besides the algebraic structures of graphs, their topological structures have been an interesting subject of research. Various topologies have been created from graphs by different methods by some researchers. In 2013, M. Amiri et. al. have created a topology using vertices of an undirected graph[3]. K.A. Abdu and A. Kılıçman have investigated the topologies generated by directed graphs in 2018 [2]. In [1], researchers have generated a topology called as incidence topology using incident vertices for each edge of a simple graph without isolated vertices. In 2020, H. K. Sarı and A. Kopuzlu have generated a topological space from a simple undirected graph without isolated vertices using vertices set of this graph [10]. They have investigated continuity and openness of functions defined on this topological space. Moreover, they have investigated condition that these topological spaces are homeomorph.

With the development of technology in recent centuries, connectedness around the world has become the most complex. There are many advantages of connectedness such as ease of global communication, the rapid spread of news and information. On the other hand, there are also its disadvantages such as the spread of epidemic disease and financial crises with surprising speed and intensity. The reason why an epidemic emerging somewhere in the world today turns into a major epidemic worldwide is this complex connectedness between countries. Connected graphs are useful tools to symbolize this connectivity in the world.

If we consider the countries of the world as the vertices of a graph, the connections between these countries as the edges of this graph, this graph becomes a giant connected graph. These connections can be economic relations, social relations, etc. From this point of view, we present a new approach to studying topological structure of a connected graph in this paper. We aim to generate a metric topology from the connected graph. We firstly define a metric on a connected graph using the length of the shortest path between two vertices. Then, we show that the metric topology generated from this metric is the discrete topology on the vertices set of this graph. Finally, we define the vertice-centered metric topology for each vertice of the graph. This metric topology will be useful to study the course of an epidemic that affects the whole world. The rest of this paper is organized as follows. In section 2, it is presented some fundamental concepts related to topological spaces and graph theory. In section 3, vertice-centered metric topology is defined on vertices set of a connected undirected graph and this metric topology is studied. In section 4, some properties of this metric topological space and the vertice-centered metric topologies generated from certain graph are investigated. This paper is concluded in section 5.

2 Preliminaries

In this section, some fundamental definitions and theorems used in the work is presented.

2.1 Graph Theory

Definition 2.1. [4] *Let U be a non-empty set of objects and E be the set of links connecting the certain unordered pairs of these objects. Then $G = (U, E)$ is called an undirected graph. U is called the set of vertices of G and E is called the set of edges of G. If both U and E are finite then G is a finite graph. If there is an edge connecting vertices v_1 and v_2, these vertices are called adjacent vertices.*

A loop is an edge linking same two vertices. There is more an edge linking same pair of vertices, these edges are called multiple edges. A graph is called simple graph, if it has no loop or multiple edges. The simple graph whose vertices are pairwise adjacent is a complete graph. A complete graph with n vertices is denoted K_n.

Definition 2.2. [4] *Let $G = (U, E)$ be a graph, where $U = \{v_i : i \in U\}$ and $E = \{e_i : i \in I\}$. A sequence $v_1e_1v_2e_2v_3e_3....v_{k-1}e_{k-1}v_k$ of the finite number of adjacent vertices and edges connecting to these vertices is called a walk. If each vertice and each edge in this sequence is used at most one times, it is called a path. If $v_1 = v_k$, the path is a cycle. A cycle with n vertices it is denoted with C_n.*

Definition 2.3. [6] *A graph $G = (U, E)$ is called a connected graph, if there is a path between its every two vertices. The number of edges in a path*

$$v_1e_1v_2e_2v_3e_3....v_{k-1}e_{k-1}v_k,$$

is called length of this path.

Definition 2.4. [6] *Let $G = (U, E)$ be a graph and v_1, v_2 be two vertices of G. The length of the shortest path between v_1 and v_2 is the distance between these two vertices.*

Definition 2.5. [6] *A graph is bipartite if its vertices set can be divided into two subsets such that every edge connects a vertice in one subset to a vertice in the other subset. (In other words, no edges join vertices that belong to the same set; all edges go between the two subsets.) A bipartite complete graph is denoted $K_{X,Y}$.*

2.2 Topological Concepts

Definition 2.6. [8]*Let U be a non-empty set and $d : U \times U \to \mathbb{R}$ be a function. Then d is called a metric on U if it satisfies the following axioms:*

1. *$d(x, y) \geq 0$, for all $x, y \in U$ and $d(x, y) = 0$ if and only if $x = y$.*
2. *$d(x, y) = d(y, x)$, for all $x, y \in U$.*
3. *$d(x, z) \leq d(x, y) + d(y, z)$, for all $x, y, z \in U$.*

Suppose $\varepsilon > 0$. $B(x, \varepsilon)$ is defined as an open sphere with center x and radius ε. Alternatively, $B(x, \varepsilon)$ is denoted as $B(x, \varepsilon) = \{y : d(x, y) < \varepsilon\}$.

The diameter of a non-empty subset X of U defined as

$$d(X) = \sup\{d(x, x') : x, x' \in X\}.$$

Theorem 2.1. [8] *Let U be a non-empty set and β be a class of subsets of U. If following conditions are satisfied, the collection β is a base for a topology.*

1. *$U = \bigcup_{B\in\beta} B$.*
2. *For $B_1 \in \beta$ and $B_2 \in \beta$, if $x \in B_1 \cap B_2$, then there is $B_3 \in \beta$ such that $x \in B_3 \subset B_1 \cap B_2$.*

The class of open spheres in a metric space (U, d) is a base for a topology on U. The topology τ on U generated by the class of open spheres in U is called metric topology.

Definition 2.7. [7] *Given a topological space (U, τ).*

1. *U is called T_1-space, if there exist $G_1, G_2 \in \tau$ such that $x \in G_1 (y \notin G_2)$ and $y \in G_2 (x \notin G_2)$, for every $x, y \in U$.*
2. *U is called Hausdorff-space, if for every $x, y \in U$, there exist $G_1, G_2 \in \tau$ such that $x \in G_1$, $y \in G_2$ and $G_1 \cap G_2$.*
3. *U is called normal space, if there exist disjoint open subsets G_1, G_2 such that $K_1 \subset G_1$ and $K_2 \subset G_2$, for each pair K_1, K_2 of disjoint closed subsets of U.*

Theorem 2.2. [7] *A topological space U is connected if and only if U is not the union of two non-empty disjoint open sets.*

3 The Vertice-centered Metric Topology of A Connected Undirected Graph

Theorem 3.1. *Given a connected undirected graph $G = (U, E)$. We define $d_G : U \times U \to \mathbb{R}$ function on U by following equation,*

$$d_G(v_i, v_j) = \begin{cases} 0, & \text{if } v_i = v_j \\ \text{Distance between } v_i \text{ and } v_j, & \text{if } v_i \neq v_j, \end{cases}$$

where $v_i, v_j \in U$. Then the function d_G is a metric on U.

Proof.

1. Let $v_i, v_j \in U$. Then $v_i = v_j$ or $v_i \neq v_j$. Suppose $v_i = v_j$, then we obtain $d_G(v_i, v_j) = 0$ from definiton of d_G. Let $v_i \neq v_j$, then $d_G(v_i, v_j)$ is distance between v_i and v_j. Because distance between two vertice of a graph is not negative, we have $d_G(v_i, v_j) > 0$. Thus it is obtained that $d_G(v_i, v_j) \geq 0$, for all $v_i, v_j \in U$. Moreover, it is clearly seen that $d_G(v_i, v_j) = 0$ if and only if $v_i = v_j$.
2. It is seen that $d_G(v_i, v_j) = d_G(v_j, v_i)$, for all $v_i, v_j \in U$ from definition of d_G .
3. Given $v_i, v_j, v_k \in U$. Then we examine the all possible cases. Let $v_i = v_j = v_k$, then we have $d_G(v_i, v_k) < d_G(v_i, v_j) + d_G(v_j, v_k)$. Let $v_i = v_j \neq v_k$, then we obtain $d_G(v_i, v_k) = d_G(v_i, v_j) + d_G(v_j, v_k)$. Similarly, in case of $v_i \neq v_j = v_k$, it is seen that $d_G(v_i, v_k) = d_G(v_i, v_j) + d_G(v_j, v_k)$. Suppose that $v_i = v_k \neq v_j$, then $d_G(v_i, v_k) = d_G(v_i, v_j) + d_G(v_j, v_k)$. Let $v_i \neq v_j$, $v_j \neq v_k$ and $v_i \neq v_k$. Let v_j is on shortest path to vertice v_k from vertice v_i. Then

$$v_i e_{a_1} v_{i+1} e_{a_2} \ldots\ldots e_{a_n} v_j e_{a_{n+1}} \ldots\ldots e_{a_m} v_k.$$

Then it is obtained that

$$d_G(v_i, v_k) = m,\ d_G(v_i, v_j) = n \text{ and } d_G(v_j, v_k) = m - n.$$

Thus, it is obtained that $d_G(v_i, v_k) = d_G(v_i, v_j) + d_G(v_j, v_k)$. Suppose that v_j is not on shortest path to vertice v_k from vertice v_i. Since v_j is not on shortest path to vertice v_k from vertice v_i, it is obtained that

$$d_G(v_i, v_k) < d_G(v_i, v_j) + d_G(v_j, v_k).$$

Consequently, for all $v_i, v_j, v_k \in U$, we have

$$d_G(v_i, v_k) \leq d_G(v_i, v_j) + d_G(v_j, v_k).$$

□

Definition 3.1. *Let $G = (U, E)$ be a connected undirected graph. Then the metric defined as above is called metric of the graph G.*

Example 3.1. *Given a connected undirected graph $G = (U, E)$ as in Figure 1.*

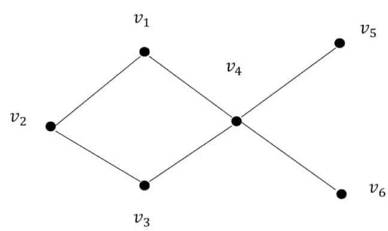

Figure 1: The Graph $G = (U, E)$

Then it is seen that $d_G(v_1, v_2) = 1, d_G(v_1, v_3) = 2, d_G(v_1, v_4) = 1$, $d_G(v_1, v_5) = 2, d_G(v_1, v_6) = 2$. Similarly, distances between other vertices can be obtained.

Definition 3.2. *Let $G = (U, E)$ be a connected undirected graph and d_G be the metric of G. Then diameter of G is defined by the equation:*

$$d_G(G) = \max\{d_G(v_i, v_j) : v_i, v_j \in U\}.$$

Theorem 3.2. *Let $G = (U, E)$ be a connected undirected graph and d_G be the metric of G. The metric topology generated by d_G is the discrete topology on U.*

Proof. The metric topology consists of arbitrary unions of the class of the open spheres $\beta_G = \{B(v_i, \varepsilon) : v_i \in U, \varepsilon > 0\}$. For every $v_i \in U$, $B(v_i, \varepsilon) = \{v_i\}$ when $0 < \varepsilon \leq 1$. Hence, it is seen that β include all singleton sets. As a result of this, we say the metric topology is discrete topology on U. □

Theorem 3.3. *Let $G = (U, E)$ be a connected undirected graph, d_G be the metric of G and $v_i \in U$. The class of*

$$\beta_{v_i} = \{B(v_i, \varepsilon) : \varepsilon > 0\},$$

is a base for a topology on U.

Proof. 1. Since G is a connected graph, there is a path to every $v_j \in U$ from v_i. Then it is seen that

$$\bigcup B(v_i, \varepsilon) = U.$$

2. Since $v_i \in B(v_i, \varepsilon_1) \cap B(v_i, \varepsilon_2)$, $B(v_i, \varepsilon_1) \cap B(v_i, \varepsilon_2) \neq \emptyset$. Let $B(v_i, \varepsilon_1), B(v_i, \varepsilon_2) \in \beta_{v_i}$ and $v_j \in B(v_i, \varepsilon_1) \cap B(v_i, \varepsilon_2)$. Then we have $d_G(v_i, v_j) < \varepsilon_1$ and $d_G(v_i, v_j) < \varepsilon_2$. Thus it is obtained that $v_j \in B(v_i, \varepsilon) \subseteq B(v_{i,}\varepsilon_1) \cap B(v_i, \varepsilon_2)$, where $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$.

Consequently, it is seen that the class β_{v_i} is a base for a topology on U. □

Definition 3.3. *Let $G = (U, E)$ be a connected undirected graph. The topology on U generated by the class β_{v_i} is called v_i-centered metric topology of the graph G. It is denoted by τ_{v_i}.*

Corollary 3.1. *Let $G = (U, E)$ be a connected undirected graph and τ_{v_i} be the v_i-centered metric topology of the graph G. Then the elements of τ_{v_i} is totally ordered by inclusion.*

It is clearly seen that while the metric topology on the graph G is discrete topology, τ_{v_i} is different for each vertice v_i. Now we give an example.

Example 3.2. *The connected graph $G = (U, E)$ is given in Figure 2, where $U = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$.*

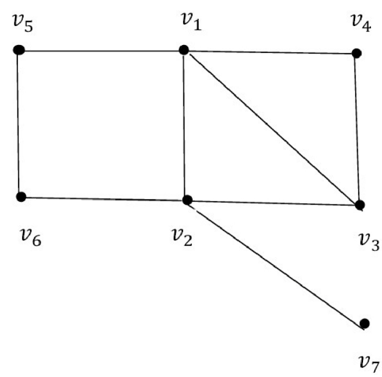

Figure 2: The Graph $G = (U, E)$

We examine the vertice-centered metric topologies of G . We obtain

$$d_G(v_1, v_2) = 1, d_G(v_1, v_3) = 1, d_G(v_1, v_4) = 1, d_G(v_1, v_5) = 1, d_G(v_1, v_6) = 2, d_G(v_1, v_7) = 2.$$

Thus, it is obtained that

$$B(v_1, \varepsilon) = \begin{cases} \{v_1\}, & \text{if } \varepsilon \leq 1, \\ \{v_1, v_2, v_3, v_4, v_5\}, & \text{if } 1 < \varepsilon \leq 2, \\ U, & \text{if } \varepsilon > 2, \end{cases}$$

and v_1-centered metric topology is as follow

$$\tau_{v_1} = \{\emptyset, U, \{v_1\}, \{v_1, v_2, v_3, v_4, v_5\}\}.$$

Similarly, v_2-centered metric topology is as follow

$$\tau_{v_2} = \{\emptyset, U, \{v_2\}, \{v_1, v_2, v_3, v_6, v_7\}\},$$

and for other vertices v_i-centered metric topologies is respectively as follows

$$\begin{aligned}
\tau_{v_3} &= \{\emptyset, U, \{v_3\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_4, v_5, v_7\}\},\\
\tau_{v_4} &= \{\emptyset, U, \{v_4\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_3, v_4, v_5\}\},\\
\tau_{v_5} &= \{\emptyset, U, \{v_5\}, \{v_1, v_5, v_6\}, \{v_1, v_2, v_3, v_4, v_5, v_6\}\},\\
\tau_{v_6} &= \{\emptyset, U, \{v_6\}, \{v_2, v_5, v_6\}, \{v_1, v_2, v_5, v_6, v_7\}\},\\
\tau_{v_7} &= \{\emptyset, U, \{v_7\}, \{v_2, v_7\}, \{v_1, v_2, v_3, v_6, v_7\}\}.
\end{aligned}$$

As a result of above example, we give following proposition.

Proposition 3.1. *The vertice-centered metric topology for each vertice of G is different.*

Proof. Given the connected undirected graph $G = (U, E)$, where $U = v_1, v_2, ..., v_n$. Consider two distinct vertices v_i, v_j of G. The bases of v_i-centered and v_j-centered metric topologies are as follows,respectively:

$$\beta_{v_i} = \{B(v_i, \varepsilon) : \varepsilon > 0\},\ \beta_{v_j} = \{B(v_j, \varepsilon) : \varepsilon > 0\}.$$

Since $B(v_i, \varepsilon) = v_i$, $B(v_j, \varepsilon) = v_j$, for $\varepsilon \leq 1$ and these metric topologies are totally ordered by inclusion, It is obtained that $\tau_{v_i} \neq \tau_{v_j}$. □

It is seen that the numbers of elements of the vertice-centered metric topologies for all vertices of the graph are not equal in above example. What determines the number of elements of these topologies is distances of the center vertice to other vertices. For instance, the vertices with the maximum distance to vertice are v_1,v_6 and v_7 and this distance is two. The number of elements of the topology τ_{v_i} is four.

Theorem 3.4. *Let $G = (U, E)$ be a connected undirected graph and $r = \max\{d_G(v_i, v_j) : v_j \in U\}$, for $v_i \in U$. Then the number of elements of v_i-centered metric topology is $r + 2$.*

Proof. Since $r = \max\{d_G(v_i, v_j) : v_j \in U\}$, it is seen that

$$B(v_i, \varepsilon) = \begin{cases} \{v_i\}, & \text{if } \varepsilon \leq 1,\\ \{v_j : d_G(v_i, v_j) < \varepsilon\}, & \text{if } 1 < \varepsilon \leq 2,\\ \cdots & \cdots\\ U, & \text{if } \varepsilon > r + 1, \end{cases}$$

$$\beta_{v_i} = \{\{v_i\}, \{v_j : d_G(v_i, v_j) < 2\}, \{v_j : d_G(v_i, v_j) < 3\}, ..., U\}.$$

Here, it is clearly seen that the number of elements of β_{v_i} is $r + 1$. Then we obtain that

$$\tau_{v_i} = \{\emptyset, \{v_i\}, \{v_j : d_G(v_i, 1) < 2\}, \{v_j : d_G(v_i, 2) < 3\}, ..., U\}.$$

Thus we say the number of elements of τ_{v_i} is $r + 2$. □

4 Some Properties of Vertice-Centered Metric Topologies

Theorem 4.1. *Given a connected undirected graph $G = (U, E)$ and the v_i-centered metric topology τ_{v_i}, where $v_i \in U$. Then the topological space (U, τ_{v_i}) is not T_1-space.*

Proof. Since the elements of τ_{v_i} is totally ordered by inclusion, there is not open subsets $V_1$and V_2 of U such that $v_i \in V_1, v_j \notin V_1$ and $v_j \in V_2, v_i \notin V_2$, for every $v_i, v_j \in U$. Therefore, (U, τ_{v_i}) is not T_1-space. Since the elements of τ_{v_i} is totally ordered by inclusion, there is not open subsets V_1 and V_2 of U such that $v_i \in V_1, v_j \notin V_1$ and $v_j \in V_2, v_i \notin V_2$, for every $v_i, v_j \in U$. Therefore, (U, τ_{v_i}) is not T_1-space. □

Corollary 4.1. *Given a connected undirected graph $G = (U, E)$ and the v_i-centered metric topology τ_{v_i}, where $v_i \in U$. Then the topological space (U, τ_{v_i}) is not Hausdorff space.*

Theorem 4.2. *Let $G = (U, E)$ be a connected undirected graph and τ_{v_i} be the v_i-centered metric topology, where $v_i \in U$. Then the topological space (U, τ_{v_i}) is a normal space.*

Proof. The v_i-centered metric topology is in the form of:

$$\tau_{v_i} = \{\emptyset, \{v_i\}, \{v_j : d_G(v_i, v_j) < 2\}, \{v_j : d_G(v_i, v_j) < 3\}, ..., U\}.$$

Suppose that $G_1 = \{v_i\}, G_2 = \{v_j : d_G(v_i, v_j) < 2\}, G_3 = \{v_j : d_G(v_i, v_j) < 3\}, ..., G_n = \{v_j : d_G(v_i, v_j) < n\}, ...$ From 3.1 it is seen that the elements of τ_{v_i} is totally ordered by inclusion. That is, for every $i, j \in I$ such that $i < j$

$$G_i \subset G_j.$$

Then, for every $i, j \in I$ such that $i < j$, it is obtained that

$$(G_j)^c \subset (G_i)^c.$$

The class of closed set of U is as follows:

$$\mathbf{K}_{v_i} = \{G_i^c : G_i \in \tau_{v_i}\}.$$

Thus, the elements of $\mathbf{K}_{v_i}$ is totally ordered by inclusion. Let K_1 and K_2 be disjoint closed subsets of (U, τ_{v_i}). Then one of them is certainly $\emptyset$. Suppose that $K_1 = \emptyset$. Then $\emptyset$ and X are disjoint open subsets of U which include K_1 and K_2, respectively. Therefore, (U, τ_{v_i}) is a normal space. □

Theorem 4.3. *Given a connected undirected graph $G = (U, E)$ and the v_i-centered metric topology τ_{v_i}, where $v_i \in U$. Then the topological space (U, τ_{v_i}) is connected.*

Proof. The base of v_i-centered metric topology τ_{v_i} is in the form:

$$\beta_{v_i} = \{B(v_i, \varepsilon) : \varepsilon > 0\}.$$

Since $v_i \in B(v_i, \varepsilon)$, for every $\varepsilon > 0$, it is seen that $G \cap H \neq \emptyset$, for every $G, H \in \tau_{v_i}$. Thus, it is seen that there are not $G, H \in \tau_{v_i}$ such that $G \cup H = U$ and $G \cap H = \emptyset$. Consequently, (U, τ_{v_i}) is connected. □

Example 4.1. *The cycle $C_n = (U, E)$ is given in Figure 3, where $U = v_1, v_2, ..., v_n$ and $n \geq 3$. We investigate the vertice-centered metric topology for each vertice of C_n. The base of v_1-centered metric topology is as follow:*

$$B(v_1, \varepsilon) = \begin{cases} \{v_1\}, & \text{if } \varepsilon \leq 1, \\ \{v_1, v_2, v_n\}, & \text{if } 1 < \varepsilon \leq 2, \\ ... & ... \\ U, & \text{if } \varepsilon > \dfrac{n}{2}. \end{cases}$$

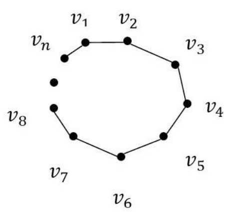

Figure 3: The Cycle $C_n = (U, E)$

When $n = 5$, the vertice-centered metric topologies of C_5 is as follows:

$$\begin{aligned}
\tau_{v_1} &= \{\emptyset, U, \{v_1\}, \{v_1, v_2, v_5\}\},\\
\tau_{v_2} &= \{\emptyset, U, \{v_2\}, \{v_1, v_2, v_3\}\},\\
\tau_{v_3} &= \{\emptyset, U, \{v_3\}, \{v_2, v_3, v_4\}\},\\
\tau_{v_4} &= \{\emptyset, U, \{v_4\}, \{v_3, v_4, v_5\}\},\\
\tau_{v_5} &= \{\emptyset, U, \{v_5\}, \{v_1, v_4, v_5\}\}.
\end{aligned}$$

Example 4.2. *We investigate vertice-centered metric topology for each vertice of $K_n = (U, E)$, where $U = v_1, v_2, ..., v_n$ and $n \geq 3$. Since the graph K_n is complete, for each $v_i \in U$, the base of the v_i-centered metric topology is as follows:*

$$B(v_i, \varepsilon) = \begin{cases} \{v_i\}, & \text{if } \varepsilon \leq 1, \\ U, & \text{if } \varepsilon > 1. \end{cases}$$

Thus, the v_i-centered topology is as follows:

$$\tau_{v_i} = \{\emptyset, U, \{v_i\}\}.$$

It is seen that the number of elements of the vertice-centered metric topology for each vertice of K_n is three.

Example 4.3. *Let us investigate the bipartite graph $K_{X,Y} = (U, E)$, where $U = v_1, v_2, ..., v_n, X = v_1, v_3, ..., v_{n-1}$ and $Y = v_2, v_4, ..., v_n$. Let $v_i \in X$,then it is obtained that*

$$d_G(v_i, v_j) = \begin{cases} 1, & \text{if } v_j \in Y, \\ 2 & \text{if } v_j \in X, \end{cases}$$

where $v_i \neq v_j$. Thus, the base of the v_i-centered metric topology is as follows:

$$B(v_i, \varepsilon) = \begin{cases} \{v_i\}, & \text{if } \varepsilon \leq 1, \\ \{\{v_i\}, \{v_j : v_j \in Y\}\}, & \text{if } 1 < \varepsilon \leq 2, \\ U, & \text{if } \varepsilon > 2. \end{cases}$$

Then, v_i-centered metric topology of $K_{X,Y}$ is as follows:

$$\tau_{v_i} = \{\emptyset, U, \{v_i\}, \{\{v_i\}, \{v_j : v_j \in Y\}\}\}.$$

Similarly, for $v_i \in Y$, v_i-centered metric topology of $K_{X,Y}$ is as follows:

$$\tau_{v_i} = \{\emptyset, U, \{v_i\}, \{\{v_i\}, \{v_j : v_j \in X\}\}\}.$$

It is seen that the number of elements of the vertice-centered metric topology for each vertice of $K_{X,Y}$ is four.

Corollary 4.2. *The number of elements of the vertice-centered metric topology for each vertice of K_n is three,for every $n \geq 3$ .The number of elements of the vertice-centered metric topology for each vertice of $K_{X,Y} = (U, E)$ is four, where $S(U) \geq 4$.*

5 Conclusion

In this paper, a metric on a connected undirected graph is defined. It is shown that the metric topology generated by this metric is a discrete topology on the vertices set of the graph. Moreover, a different topology for each vertice of a connected undirected graph is built using this metric. Several properties of these topologies called as the vertice-centered metric topologies are studied. It is investigated the vertice-centered metric topologies generated from certain graphs.

Conflicts of Interest The authors declare that there is no conflict of interest in this article.

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